CSC 202 midterm ‘open’ notes:

^ : and

ˇ : or

¬: not

->: if p then q

<->: p if and only if q

U is union

∩ is intersection

- is difference

\*three equals sign is logically equivalent to

tautology: always true

falsehood: always false

disjunctive normal form: use truth table connect true parts ^ inside clauses ˇ connecting clauses

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**Reflexive**: R(x, x) for all x

**Symmetric**: R(x,y) -> R(y,x)

**Transitive**: R(x,y) ^ R(y,z) -> R(x,z)

**Anti-symmetric**: R(x,y) ^ R(y,x) -> x = y

**Anti-Reflexive**: ¬(R(x,x)) for all x

**Equivalence relation**: if it is reflexive, symmetric and transitive

**Ordering relation**: if it is reflexive, anti-symmetric and transitive

A={x:x A^x A}.But an element cannot both be in the set A and not be in the set A. So {x : x A^x A} must be empty.

P(P(0)) = P({0}) = {0, {0}} *(0 = empty set)*

0 1 0 0 0 0

0 1 0 When this is flipped over the diagonal it becomes: 1 1 1

0 1 0 0 0 0

- makes it neither reflexive nor symmetric

- reflexive is the diagonal, symmetric is if it’s the same after being flipped over the diagonal.

**Problem 10:**

10) Determine if the relation R(x,y) = “x = y + 1 or x = y – 1” is reflexive, anti-reflexive,

symmetric, anti-symmetric and/or transitive. Either prove that it has a given property.

**Solution**:

R(x,y) = “x = y + 1 or x = y – 1” is not reflexive since x != x+1 and x != x -1 for all integers x.

R(x,y) = “x = y + 1 or x = y – 1” is anti-reflexive since x != x+1 and x!= x -1 for all integers x.

R(x,y) = “x = y + 1 or x = y – 1” is symmetric since if x = y+1 or x = y -1, then y = x – 1 or y = x + 1 using basic algebra.

R(x,y) = “x = y + 1 or x = y – 1” is not anti-symmetric. Suppose x = y+1 or x = y-1 and y = x+1 or y = x-1. There are four cases:

1. x = y + 1 and y = x + 1. This isn’t possible since it would mean that x = y + 1 = (x + 1) + 1 = x + 2.

2. x = y + 1 and y = x – 1. While this is possible, it does mean that x != y.

3. x=y–1andy=x + 1. Again this is possible, but it again means that x != y.

4. x=y–1andy=x – 1. This isn’t possible since it would mean that x = y – 1 = (x

– 1) – 1 = x – 2.

R(x,y) = “x = y + 1 or x = y – 1” is not transitive since for x = 3, y = 2, and z = 1, we have that x = y – 1 and y = z – 1 but x != z + 1 and x != z – 1.

**14)** Write a negation for the following statements:

a)( x) [if x(x+1) > 0 then x > 0 or x < -1] quantified over the real numbers

b)( x) ( y) ( z) [if x – y is even and b – c is even, then a – c is even]

**Solution:**

a) ( x)[x(x+1) > 0 ^ x =< 0 ^ x ≥ -1]

b) ( x) ( y) ( z) [x – y is even and b – c is even, but a – c is odd]

**15)**

Indicate whether the following statements are true or false where the statements are quantified over the real numbers. If the statement is false, indicate clearly why the statement is false. If the statement is true, indicate clearly why the statement is true.

a)( x) ( y)[x + y = 0]

b)( y) ( x) [x + y = 0]

**Solution**:

a) True. If you choose –x, then x + (-x) = 0.

b) False. There is no real number that when added to all real numbers, produces 0.